explicit form of functions of the parameters, and the Dorodnitsyn variables (1) must be replaced by the "physical" variables x and r.

We note that the theoretical analysis of similarity of flow of a viscous gas in pipes with heat exchange is analyzed theoretically, e.g., in [4].

NOTATION

x, r, cylindrical coordinates; u, v, velocity components; p_W, ρ_W, and ν_W, pressure, density, and kinematic coefficient of viscosity for temperature of the wall; $\bar{v} = v \cdot H^{-1} + \frac{\partial \eta}{\partial x} u$; $\Delta = \int_0^{r_0(x)} H^{-1} dr$; $\bar{r} = \int_0^{\varphi} H d\varphi$; $Q = \int_0^{\Delta} r u d\eta = \frac{Q_0}{2\pi\rho_w}$; Q_0, constant flow rate of the gas.

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CALCULATION OF THE LAMINAR FLOW OF AN INCOMPRESSIBLE LIQUID AROUND A DISC AND A CYLINDER

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We study the circular flow of a viscous incompressible liquid around a disc and a cylinder, in the range of Reynolds numbers $40 \leq \mathrm{Re} \leq 1000$.

The aim of the present work is to obtain stable and sufficiently accurate numerical solutions for the flow near a disc and a cylinder. The bodies are immersed in a circular flow of a viscous incompressible liquid, with a zero angle of incidence. This type of information is essential since, in the construction of models of flow of a liquid which contains solid particles, one usually uses the data about the action of the liquid on an individual particle [1]. The solution is based on the difference approximation of the Navier–Stokes equations according to a scheme used in [2]. There are a number of features of the scheme that make it useful for the study of the flow considered here, which is characterized by the presence of developed circulation zones. These features are: the use of velocity components and pressure correction as the basic independent variables, the displacement of the grid for velocity components, the combination of unilateral and central difference in the approximation of convection terms (hybrid scheme). The solution is limited to the region of Reynolds numbers constructed from the unperturbed flow and from the diameter of the disc or cylinder, i.e., $40 \leq \mathrm{Re} \leq 1000$. Possible effects of three-dimensionality of the flow were not considered.

In a cylindrical coordinate system (x, r), the equation for the change of momentum and the continuity equation can be written in the form

$$\frac{1}{r}\left[\frac{\partial}{\partial x}(urf) + \frac{\partial}{\partial r}(vrf) - \frac{\partial}{\partial x}\left(r\Gamma_f \frac{\partial f}{\partial x}\right) - \frac{\partial}{\partial r}\left(r\Gamma_f \frac{\partial f}{\partial r}\right)\right] = S_f. \quad (1)$$

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TABLE 1. Dependence of the Resistance Coefficients on Re

L	Re	c_{xp}	c_{xf}	c_{xd}
0 (disc)	40	1,85	—	0,800
	100	1,36	—	0,476
	250	1,08	—	0,209
	500	1,03	—	0,144
	1000	1,02	—	0,091
1	40	1,41	0,891	0,354
	100	1,08	0,368	0,235
	250	0,950	0,098	0,156
	500	0,899	0	0,115
	1000	0,913	—0,058	0,093
1,5	40	1,38	1,17	0,294
	100	1,05	0,502	0,209
	250	0,934	0,168	0,135
	500	0,882	0,037	0,103
	1000	0,890	—0,042	0,081
2	40	1,36	1,41	0,266
	100	1,03	0,629	0,194
	250	0,929	0,235	0,122
	500	0,875	0,068	0,098
	1000	0,883	—0,026	0,080

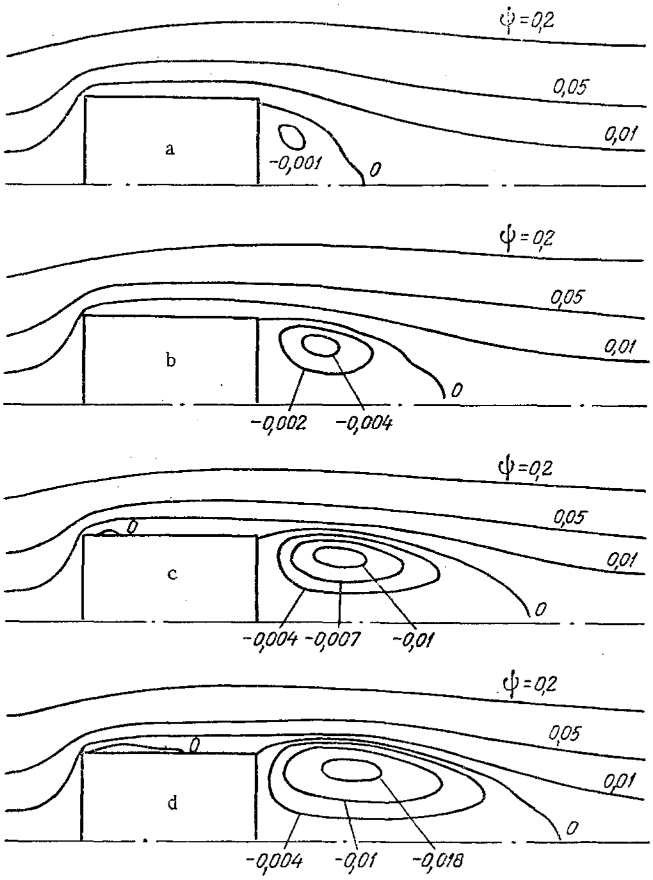

Fig. 1. Lines of constant values of the flow function ψ for a cylinder of length L=1, for Reynolds numbers Re=40 (a), 200 (b), 250 (c), and 500 (d).

In the equation for momentum, f replaces u or v. In this case $\Gamma_u = \Gamma_v = 1/\mathrm{Re}$; $S_u = -\partial P/\partial x$; and $S_v = -v/(r^2\mathrm{Re}) - \partial P/\partial r$. In the continuity equation f=1; and $\Gamma_f = S_f = 0$.

In the difference approximation of Eqs. (1), a rectangular grid is constructed in such a way that the boundaries of the region of calculation which includes the surface of the body and the symmetry axis pass between coordinate lines. The intersections of the coordinate lines of the grid form "principal" sites where one calculates the pressure and pressure correction. The pressure correction is taken as the principal variable, and is defined as the difference of pressures at two consecutive computational steps. The sites where one calculates the velocity components are positioned midway between the principal sites; the sites with the axial velocity u are displaced in the axial direction, and the sites with the radial velocity v in the radial direction. The finite-difference equations are obtained by integrating Eq. (1) over control volumes associated with the sites in which one calculates the pressure correction and velocity components. In the calculation of flows through the sides of the control volumes, we used the so-called hybrid scheme, according to which the difference approximation depends on the Peclét number of the grid for the side of the control volume under investigation. If $|Pe| < 2$, one applies central differences, for convective terms, and diffusion terms are retained; if $|Pe| \geq 2$, one applies differences against the flow for convective terms, and diffusive terms are omitted. This approach ensures the stability of the numerical procedure, as the application of central differences for $|Pe| \geq 2$ leads to the result that the Courant iteration number becomes larger than unity, and the solution becomes unstable.

Thus, for some site 0 we have the finite-difference equation

$$A_0 f_0 = \sum_{i=1}^{4} A_i f_i + B_0, \tag{2}$$

where $f \equiv u, v, p$, and the summation is carried out over the four nearest neighbors of 0.

After the determination of the pressure correction, one calculates the pressure and the more accurate values of the velocity components:

$$P_0 = P_0^* + p_0, \tag{3}$$

$$f_0 = f_0^* - C_0 \Delta p, \tag{4}$$

where $f \equiv u, v$; Δp is the difference of values of p at two principal sites neighboring with the site at which one improves the accuracy of corresponding velocity component. The derivation of Eqs. (2)-(4) is discussed in more detail in [2].

Let us consider the boundary conditions. At the symmetry axis we take $v = \partial u/\partial r = \partial P/\partial r = \partial p/\partial r = 0$.

At the surface of the body we use the condition of adhesion of the liquid, i.e., $u = v = 0$. The condition of vanishing component of velocity normal to the body is specified directly in the calculation. The equality to zero of the tangential component is achieved by a method usual in numerical calculations, by introducing velocity at a fictitious site inside the body. In addition, at the surface of the body we specify that the normal gradient of p is equal to zero (this follows from (4)),

At the external front (with respect to the stream) and upper boundaries of the region of calculations we specify conditions in the unperturbed flow: $u = 1$ and $v = P = p = 0$. At external rear boundary we specify "soft" conditions for the axial velocity component, and conditions in the unperturbed flow for the remaining variables: $\partial u/\partial x = v = P = p = 0$.

The initial state of the flow in a large part of the region of calculation for Re = 40 is defined using the conditions in the unperturbed flow ($u = 1$), and only in the wake of the body, we use the condition of rest ($u = 0$). At large Reynolds numbers, the initial conditions were the results of solution of the previous variant with respect to Re.

The computational algorithm consists of a sequence of operations (the external iteration cycle):

1) For given initial conditions, one solves Eq. (2) for $f = u, v$.

2) One finds the iteration solution of Eq. (2) for $f = p$ in such a way that the difference of p at two consecutive iterations is smaller than a given quantity (the internal iteration cycle).

3) One determines the pressure and more accurate values of velocity components from (3) and (4).

4) New values are used for the starting variables and the procedure is repeated.

The criterion of convergence of the external iteration cycle was chosen to be the condition of constant, with a given accuracy, resistance coefficient to the motion of the body. In the calculations, we also controlled the condition of mass conservation at the vertical lines of the grid to avoid unphysical solutions.

As is known, the accuracy and convergence of the calculation are determined by the method of solution of the system of equations (2). In the present calculation, we used the driving method which was constructed using the algorithm for inversion of a tridiagonal matrix. Following the example of the matrix used in [3] we assume that the only unknowns in (2) are the values of the variable f at the vertical line of the grid under consideration.

The values of the variable at the neighboring vertical lines are assumed known from the previous iteration step. Equation (2) can then be rewritten in the form

$$f_j = A_j f_{j+1} + B_j f_{j-1} + C_j,$$

where $f \equiv u, v, p$; $j = 2, 3, \ldots, n-1$.

To calculate the variables, the last equation will be used in the form

$$f_j = a_j f_{j+1} + b_j, \tag{5}$$

where $a_j = A_j T_j$; $b_j = (B_j b_{j-1} + C_j) T_j$; $T_j = 1/(1 - B_j a_{j-1})$; $a_2 = A_2$; $b_2 = B_2 f_1 + C_2$. We note that Eq. (5) for $f = u, v$ is solved using the method of lower relaxation, to ensure the convergence of the computational method.

The calculation is carried out for a disc of length $L = 0$ and for a cylinder of lengths $L = 1$, 1.5, and 2 (here and below, the diameter of the disc or cylinder is taken as the unit of length) for Reynolds numbers Re = 40, 100, 250, 500, and 1000. We used the following constant parameters of calculation: the coefficient of lower relaxation in (5) ($f = u, v$) equal to 0.3; the accuracy of determination of the pressure correction equal to 0.01; the accuracy of solution of the problem determined from the change of the resistance coefficient per iteration step of the external cycle equal to 0.001; and the minimum steps of the grid in axial and radial directions which were 0.01 and 0.02, respectively. The distance from the surface of the body to the front, upper, and rear external boundaries of the calculation region was taken as 7, 6, and 12 units. The maximum size of the grid 72×25. The grid is nonuniform, and is constructed in such a way that the ratio of two neighboring steps in x and r directions does not exceed 1.5. The calculation of one variant on the computer ES-1050 was ≈30 min for Re = 40. As Re increased, a considerably shorter machine time was required (on the order of 10-15 min for Re = 1000). The maximum number of iterations steps in the external iteration cycle varied from 200 to 400. The number of iterations for determination of the pressure correction (inner cycle) decreased with increasing Re. However, it remained practically constant within one variant for Re = const.

Table 1 shows the dependence of the resistance coefficients of pressure C_{xp}, friction C_{xf}, and of the bottom C_{xd} (obtained in the usual way by dividing the total pressure and friction forces by the velocity thrust of the unperturbed flow, and by the area of the middle section of the body) on the Reynolds number Re. It is seen from Table 1 that, within the limits of cylinder lengths considered here, C_{xp} depends weakly on L for all Reynolds numbers used in the calculation. The pressure resistance of the disc exceeds the pressure resistance of the cylinder due to the considerably large rarefaction in the wake of the disc, as well as to the increased pressure on its front side. It is interesting to note that the resistance coefficient of friction of the cylinder, starting from some value of the Reynolds number, becomes negative which indicates flow separation at its side surface. Naturally, this effect is more pronounced for shorter cylinders. An important characteristic is the bottom resistance coefficient which is also given in Table 1. We note that, for a constant Reynolds number, C_{xd} attains its maximum value for the disc. For long cylinders (of the order of 1.5-2), C_{xd} tends to a number constant for each Re. For these cylinder lengths, the bottom resistance consists of a fraction of the total resistance of pressure of the cylinder which varies from 19% for Re = 40 to 9% for Re = 1000.

Figure 1 shows the lines of constant values of the flow function ψ for a cylinder of length $L = 1$ for Re = 40, 100, 250, and 500. Figure 2 gives the same quantity for a disc and cylinders of lengths considered here, for Re = 1000 (the figures were constructed on the graph plotter ES-7054). It follows from Figs. 1 and 2 that the intensity of the circulation zone in the wake of the body is smaller for longer bodies at identical Reynolds numbers, and increases with increasing Re. The flow separation on the side surface of the cylinder starts for all lengths considered here already at Re = 250, and the separation point is not at the front edge of the cylinder, but is displaced further downstream. Closing of the circulation zones in the wake of the cylinder and at its side surface into a single zone takes place for Re = 1000 only for the cylinder of length $L = 1$. Finally, Fig. 3 gives the comparison of the results calculated in the present work with those given in [4]. It gives the distribution of pressure P_w (relative to twice the velocity thrust of the unperturbed flow) at the surface of the cylinder of length $L = 1.5$ for Reynolds numbers 40 and 100. From the analysis of the results one can reach the conclusion that, although the character of the pressure distribution at the surface of the cylinder is similar in both cases, the results of the present work are preferable. The strong smoothing out of the curves of the pressure distribution in the neighborhood of the angular edges of the cylinder obtained in [4] is clearly a consequence of the insufficient conservativity of the splitting method used in [4].

NOTATION

x, r, axial and radial coordinates; u, v, axial and radial velocity components; P, pressure; p, correction to pressure; f, dependent variable; ψ, flow function; Re, Reynolds number; Pe, Peclet number; L, length of the

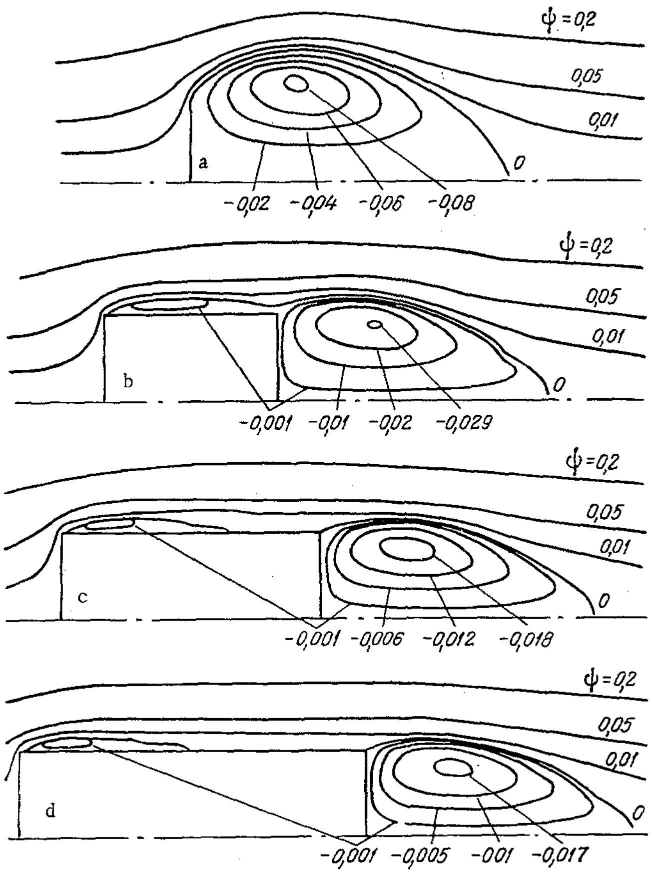

Fig. 2. Lines of constant values of the flow function ψ for a disc of length L = 0 (a) and a cylinder of length L = 1 (b), 1.5 (c), 2 (d) for Re = 1000.

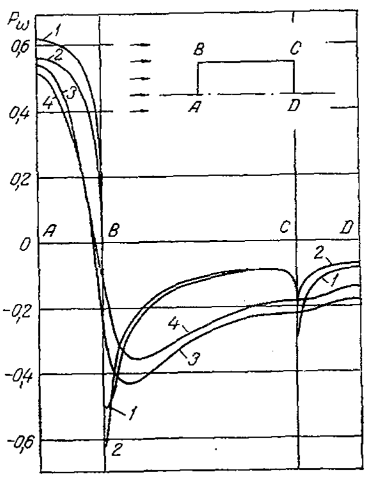

Fig. 3. Pressure distribution P_W at the surface of the cylinder of length L=1.5 for Reynolds numbers 40 (curves 1 and 3) and 1000 (curves 2 and 4). Curves 1 and 2 are the results of the present work, and curves 3 and 4 are the results of the calculation of [4] by the splitting method.

cylinder; Γ_f, exchange coefficient; S_f, source term; C_{xp}, C_{xf}, and C_{xd}, pressure, friction, and bottom resistance coefficients; n, number of nodes onthe vertical grid line; and A, B, C, a, b, coefficients in the finite-difference equations. The asterisk denotes the value of the variable at the previous step of the calculation, the subscript w denotes the value at the wall, and subscripts i, j, and 0 denote the grid nodes.

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